Oxford Cambridge and RSA

AS Level Mathematics B (MEI)

H630/02 Pure Mathematics and Statistics
Question Paper

Wednesday 23 May 2018 – Morning
Time allowed: 1 hour 30 minutes

You must have:
- Printed Answer Booklet

You may use:
- a scientific or graphical calculator

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Model Answers

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION
- The total number of marks for this paper is **70**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **12** pages. The Question Paper consists of **8** pages.

Formulae AS Level Mathematics B (MEI) (H630)

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + {}^nC_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^nC_r = {}_nC_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$$

Differentiation from first principles

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Sample variance

$$s^2 = \frac{1}{n-1}S_{xx} \text{ where } S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n} = \Sigma x_i^2 - n\bar{x}^2$$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = r) = {}^nC_r p^r q^{n-r}$ where $q = 1 - p$

Mean of X is np

Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Answer **all** the questions

1 Write down the value of

(A) $\log_a (a^4)$ **[1]**

1 A) $\log_a a^4 = 4\log_a a$
$= 4$

[1]

(B) $\log_a\left(\frac{1}{a}\right)$.

B) $\log_a \frac{1}{a} = \log_a a^{-1} = -\log_a a$
$= -1$

2 Doug has a list of times taken by competitors in a 'fun run'. He has grouped the data and calculated the frequency densities in order to draw a histogram to represent the information. Some of the data are presented in Fig. 2.

Time in minutes	15 –	20 –	25 –	35 –	45 – 60
Number of runners	12	23	59	71	
Frequency density	2.4		5.9	7.1	1.4

Fig. 2

(i) Write down the missing values in the copy of Fig. 2 in the Printed Answer Booklet. **[2]**

2 i) Number of runners : $15 \times 1.4 = 21$
Frequency density : $\frac{23}{5} = 4.6$

(ii) Doug labels the horizontal axis on the histogram 'time in minutes' and the vertical axis 'number of minutes per runner'. State which one of these labels is incorrect and write down a correct version. **[1]**

ii) The vertical axis is wrong, it should be 'number of runners per minute'

3 P and Q are consecutive **odd** positive integers such that $P > Q$.

Prove that $P^2 - Q^2$ is a multiple of 8. **[3]**

3. let $P = 2n + 1$, then $Q = 2n - 1$

$P^2 - Q^2 = (2n+1)^2 - (2n-1)^2$
$= 4n^2 + 4n + 1 - 4n^2 + 4n - 1$
$= 8n$

hence $P^2 - Q^2$ is a multiple of 8

4 The probability distribution of the discrete random variable X is given in Fig. 4.

r	0	1	2	3	4
$P(X = r)$	0.2	0.15	0.3	k	0.25

Fig. 4

(i) Find the value of k. **[2]**

X_1 and X_2 are two independent values of X.

[3]

4. i) total probabilities = 1
$0.2 + 0.15 + 0.3 + k + 0.25 = 1$
$k + 0.9 = 1$
$k = 0.1$

(ii) Find $P(X_1 + X_2 = 6)$.

ii) Three options : $X_1 = 2,\ X_2 = 4$
$X_1 = 4,\ X_2 = 2$
$X_1 = 3,\ X_2 = 3$

$P(X_1 + X_2 = 6) = 0.3 \times 0.25 + 0.3 \times 0.25 + 0.1 \times 0.1$
$= 0.075 + 0.075 + 0.01$
$= 0.16$

5 Find the set of values of a for which the equation

$$ax^2 + 8x + 2 = 0$$

has no real roots. **[3]**

5. $ax^2 + 8x + 2 = 0$

If it has no real roots, the discrimiant < 0

$b^2 - 4ac < 0$
$8^2 - 4(a)(2) < 0$
$64 - 8a < 0$
$64 < 8a$
$a > 8$

6 Show that $\int_0^9 (3 + 4\sqrt{x})\,dx = 99$. **[4]**

6. $\int_0^9 3 + 4\sqrt{x}\ dx = \left[3x + \frac{8}{3}x^{\frac{3}{2}}\right]_0^9$
$= 3(9) + \frac{8}{3}(9)^{\frac{3}{2}}$
$= 27 + \frac{8}{3}(3)^3$
$= 27 + 72$
$= 99$

7 Rose and Emma each wear a device that records the number of steps they take in a day. All the results for a 7-day period are given in Fig. 7.

Day	1	2	3	4	5	6	7
Rose	10014	11262	10149	9361	9708	9921	10369
Emma	9204	9913	8741	10015	10261	7391	10856

Fig. 7

The 7-day mean is the mean number of steps taken in the last 7 days. The 7-day mean for Rose is 10112.

(i) Calculate the 7-day mean for Emma. **[1]**

At the end of day 8 a new 7-day mean is calculated by including the number of steps taken on day 8 and omitting the number of steps taken on day 1. On day 8 Rose takes 10259 steps.

[4]

7 i) $\Sigma = 66381$

mean $= \frac{66381}{7} = 9483$

At the end of day 8 a new 7-day mean is calculated by including the number of steps taken on day 8 and omitting the number of steps taken on day 1. On day 8 Rose takes 10259 steps.

(ii) Determine the number of steps Emma must take on day 8 so that her 7-day mean at the end of day 8 is the same as for Rose.

ii) Sum of Rose's steps $= 7 \times 10112 + 10259 - 10014$

$= 71029$

The sum of Emma's steps must be the same :

$66381 - 9204 + x = 71029$

$x = 13852$

Emma needs to do 13852 steps

[4]

In fact, over a long period of time, the mean of the number of steps per day that Emma takes is 10 341 and the standard deviation is 948.

(iii) Determine whether the number of steps Emma needs to take on day 8 so that her 7-day mean is the same as that for Rose in part **(ii)** is unusually high. **[3]**

iii) $10341 + 2(948) = 12237$

$13852 > 12237$ so 13852 is an outlier, she would need to take an unusually high number of steps on day 8

8 **In this question you must show detailed reasoning.**

The centre of a circle C is at the point (−1, 3) and C passes through the point (1, −1). The straight line L passes through the points (1, 9) and (4, 3). Show that L is a tangent to C. **[7]**

8. The circle has equation $(x+1)^2 + (y-3)^2 = r^2$

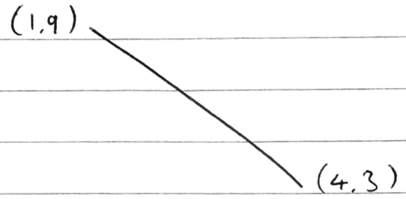
Sub in the values at the point $(1, -1)$ to find r

$r^2 = (x+1)^2 + (y-3)^2$

$r^2 = (1+1)^2 + (-1-3)^2$

$r^2 = 2^2 + (-4)^2$

$r^2 = 20$



gradient of line $= \dfrac{9-3}{1-4}$

$= \dfrac{6}{-3}$

$= -2$

Equation of line : $y - 9 = -2(x - 1)$

$y - 9 = -2x + 2$

$y = 11 - 2x$

sub this into the equation for the circle to find out where they meet

$(x+1)^2 + (11 - 2x - 3)^2 = 20$

$x^2 + 2x + 1 + (8 - 2x)^2 = 20$

$x^2 + 2x + 1 + 64 - 32x + 4x^2 = 20$

$5x^2 - 30x + 45 = 0$

$x^2 - 6x + 9 = 0$

$(x - 3)^2 = 0$

There is only one solution to this equation, so the circle and then line only meet at one point. Therefore it is a tangent to the circle

9 In this question you must show detailed reasoning.

Research showed that in May 2017 62% of adults over 65 years of age in the UK used a certain online social media platform. Later in 2017 it was believed that this proportion had increased. In December 2017 a random sample of 59 adults over 65 years of age in the UK was collected. It was found that 46 of the 59 adults used this online social media platform.

Use a suitable hypothesis test to determine whether there is evidence at the 1% level to suggest that the proportion of adults over 65 in the UK who used this online social media platform had increased from May 2017 to December 2017. **[7]**

9. let p be the proportion of over 65s who use the social media platform

$H_0 : p = 0.62$
$H_1 : p > 0.62$

$X \sim B(59, 0.62)$

$P(X \geq 46) = 1 - P(X \leq 45)$
$= 1 - 0.9932$
$= 0.0068$

$0.0068 < 0.01$, reject H_0
Sufficient evidence to suggest that the proportion of adult over 65 using the platform has increased

10 **(i)** A curve has equation $y = 16x + \frac{1}{x^2}$. Find

(A) $\frac{dy}{dx}$ **[2]**

10 i) A) $y = 16x + x^{-2}$

$\frac{dy}{dx} = 16 - 2x^{-3} = 16 - \frac{2}{x^3}$

,

[5]

(B) $\frac{d^2y}{dx^2}$.

B) $\frac{d^2y}{dx^2} = 6x^{-4} = \frac{6}{x^4}$

(ii) Hence

- find the coordinates of the stationary point,
- determine the nature of the stationary point.

ii) To find the stationary point set $\frac{dy}{dx} = 0$

$0 = 16 - \frac{2}{x^3}$

$\frac{2}{x^3} = 16$

$x^3 = \frac{1}{8}$

$x = \frac{1}{2}$

at $x = \frac{1}{2}$, $y = 16(\frac{1}{2}) + 2^2 = 8 + 4$

$= 12$

So the stationary point has coordinates $(\frac{1}{2}, 12)$

At $x = \frac{1}{2}$, $\frac{d^2y}{dx^2} = \frac{6}{(\frac{1}{2})^4} = 6(16) = 96$

$96 > 0$ so this stationary point is a minimum

11 The pre-release material contains data concerning the death rate per thousand people and the birth rate per thousand people in all the countries of the world. The diagram in Fig. 11.1 was generated using a spreadsheet and summarises the birth rates for all the countries in Africa.

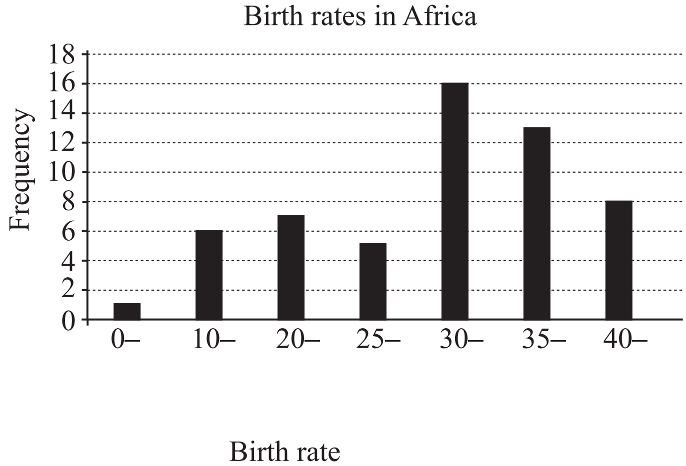


Fig. 11.1

(i) Identify **two** respects in which the presentation of the data is incorrect. **[2]**

11 i) The classes of different widths have bars of the same width

Vertical axis should be frequency density not frequency

Fig. 11.2 shows a scatter diagram of death rate, y, against birth rate, x, for a sample of 55 countries, all of which are in Africa. A line of best fit has also been drawn.

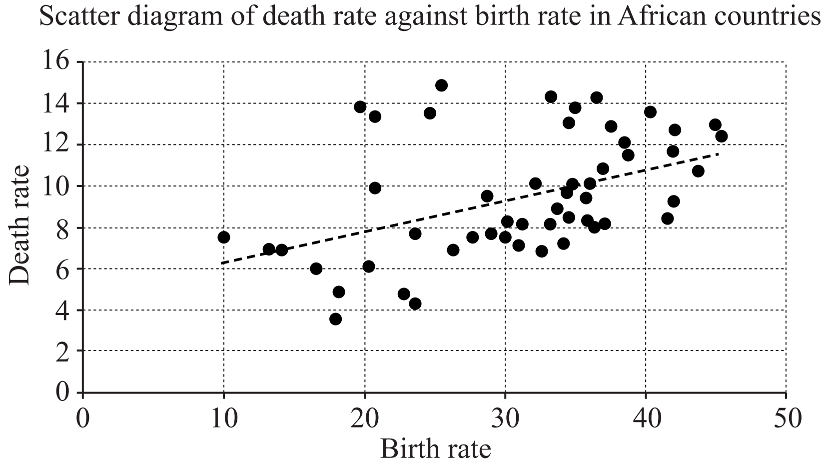


Fig. 11.2

The equation of the line of best fit is $y = 0.15x + 4.72$.

(ii) (*A*) What does the diagram suggest about the relationship between death rate and birth rate? **[1]**

ii) A) As birth rate increases, death rate increases
(positive correlation)

(B) The birth rate in Togo is recorded as 34.13 per thousand, but the data on death rate has been lost. Use the equation of the line of best fit to estimate the death rate in Togo.

B) $y = 0.15(34.13) + 4.72$
$= 9.8395$

(C) Explain why it would not be sensible to use the equation of the line of best fit to estimate the death rate in a country where the birth rate is 5.5 per thousand. **[1]**

C) You would be extrapolating

(D) Explain why it would not be sensible to use the equation of the line of best fit to estimate the death rate in a Caribbean country where the birth rate is known. **[1]**

D) The line of best fit was based off data about African countries. Birth rates and death rates may be differert in the Caribbean.

(*E*) Explain why it is unlikely that the sample is random.

E) A random sample of countries would most likely include countries from other continents

Including Togo there were 56 items available for selection.

(iii) Describe how a sample of size 14 from this data could be generated for further analysis using systematic sampling. **[2]**

iii) Generate a random number n between 1 and 4 (56 ÷ 14) and choose the n^{th} country in the list of data.
Then pick every 4^{th} country on the list, stopping when you have picked 14

12 In an experiment 500 fruit flies were released into a controlled environment. After 10 days there were 650 fruit flies present.

Munirah believes that N, the number of fruit flies present at time t days after the fruit flies are released, will increase at the rate of 4.4% per day. She proposes that the situation is modelled by the formula $N = Ak^t$.

(i) Write down the values of A and k. **[2]**

12 i) A = inital number = 500
k = growth = 1.044

[1]

(ii) Determine whether the model is consistent with the value of N at $t = 10$. [2]

ii) At $t = 10$, $N = 500(1.044)^{10}$
$= 769$

769 is not close to 650 so the model is not consistent at $t = 10$

(iii) What does the model suggest about the number of fruit flies in the long run? [1]

iii) It increases forever (exponentially)

Subsequently it is found that for large values of t the number of fruit flies in the controlled environment oscillates about 750. It is also found that as t increases the oscillations decrease in magnitude.

Munirah proposes a second model in the light of this new information.

$$N = 750 - 250\times e^{-0.092t}.$$

(iv) Identify three ways in which this second model is consistent with the known data. [3]

iv) At $t = 0$, $N = 750$
At $t = 10$, $N = 750 - 250e^{-0.92}$
$= 650.37 \approx 650$
As $t \to \infty$, $N \to 750$

(v) (A) Identify one feature which is not accounted for by the second model.

v) A) Oscillations

[1]

(B) Give an example of a mathematical function which needs to be incorporated in the model to account for this feature. [1]

B) Sine or cosine

END OF QUESTION PAPER